A Classification of Crystalline Models

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Abstract

The present study examines the classification of crystal models using symmetry. Ceramic and wooden models were classified into the 32 crystal classes and into the 6 crystal systems.

Mineralogy

The external morphology of crystals, specifically the symmetry, is a result of their internal structure. The symmetry present in the arrangement of the atoms and molecules that compose the substance will give rise to the symmetry of the crystal faces that the mineral exhibits. The symmetry of crystals are mirror planes, axes of rotation, centers of inversion, and axes of rotoinversion (Klein and Hurlbut. 1977.)

A mirror plane is a plane through a crystal where one part of the crystal is reflected through the plane to give a mirror image. A mirror plane is usually denoted as "m". A center of inversion will take all points on a crystal and invert them through the central point of the crystal. A center of inversion is usually denoted as "$\bar{1}$". These two operations give images that are not superimposable. An axis of rotation is a line about which the crystal can be rotated to give an identical appearance. A rotational axis is classified by the number of times an identical appearance occurs when the crystal is rotated 360°. The possible axes of rotation in a crystal are limited by geometry to 1,2,3,4 or 6 fold axes. An axis of rotation is

usually denoted by the letter A along with a number representing the resulting number of identical imaged generated, such as "A_2". A rotoinversion axis is a combination of a rotational axis and an inversion point which lies on the axis. A 3-fold rotoinversion would rotate a crystal face 120° and invert the face through a center of inversion. An axis of rotoinversion is usually denoted as the letter A along with a number with a line over it representing the rotational axis, such as "$A\bar{3}$". Odd-fold rotational axes can be described by a combination of other operations, such as a $A\bar{3}$ is equivalent to $\bar{1}$ and A_3. The only unique rotoinversion operation is $A\bar{4}$.

These various operations can be arranged to give 32 possible combinations of symmetry elements. Crystals can therefore be grouped into 32 classes based on the symmetry elements present. The crystal classes can be further grouped according to a set of coordinate axes which best display the crystal's symmetry. Traditionally, six crystal systems are assigned. Triclinic crystals have three unequal axes all intersecting at unequal angles. Monoclinic crystals have three unequal axes, two of which are oblique to each other and the third perpendicular to the plane of the other two. Orthorhombic crystals have three mutually perpendicular axes, all of different lengths. Tetragonal crystals have three mutually perpendicular axes, two of which are of the same length. Hexagonal crystals have four axes, three of which lie in the same plane at 120° and have the fourth perpendicular to the plane of the other three. Isometric crystals have three mutually perpendicular axes of equal lengths (Klein and Hurlbut. 1977). The total overall grouping is shown in Table 1.

Method

For this work, 2 large boxes of unsorted crystal models were provided by Dr. Tettenhorst. One of the boxes contained large wooden models and the other contained small white ceramic models. All of the white models were first classified according to crystal class and then to crystal system. A name was given to the model if it could be determined. A crystalline form name, such as cube or ditrigonal pyramid was used if possible. If the model was of a common mineral crystal, such as a quartz crystal, that name was used. For some models, no name could be determined through either method, so they were assumed to be rarer minerals. The results are shown in Tables 2-15. Where the model# is noted as "0", the model had no number stamped on it. Where the name is left blank, no name could be given to the shape. The # of models category for the ceramic models represents the number of identically shaped and sized modes of that type that were represented.

Tables

Table 1.

Crystal System	Crystal Class	Symmetry Content
Triclinic	1	none
	$\bar{1}$	$\bar{1}$
Monoclinic	2	$1A_2$
	m	m
	2/m	$\bar{1}$,$1A_2$,1m
Orthorhombic	222	$3A_2$
	mm2	$1A_2$,2m
	2/m2/m2/m	$\bar{1}$,$3A_2$,3m
Tetragonal	4	$1A_4$
	$\bar{4}$	$1A_{\bar{4}}$
	4/m	$\bar{1}$,$1A_4$,m
	422	$1A_4$,$4A_2$
	4mm	$1A_4$,4m
	$\bar{4}$ 2m	$1A_{\bar{4}}$,$2A_2$,2m
	4/m2/m2/m	$\bar{1}$,$1A_4$,$4A_2$,5m
Hexagonal	3	$1A_3$
	$\bar{3}$	$1A_{\bar{3}}$(=$\bar{1}$+$1A_3$)
	32	$1A_3$,$3A_2$
	3m	$1A_3$,3m
	$\bar{3}$ 2/m	$1A_{\bar{3}}$,$3A_2$,3m
	6	$1A_6$
	$\bar{6}$	$1A_{\bar{6}}$(=$1A_6$+m)
	6/m	$\bar{1}$,$1A_6$,1m
	622	$1A_6$,$6A_2$
	6mm	$1A_6$,6m
	$\bar{6}$m2	$1A_{\bar{6}}$,$3A_2$,3m
	6/m2/m2/m	$\bar{1}$,$1A_6$,$6A_2$,7m
Isometric	23	$3A_2$,$4A_3$
	2/m $\bar{3}$	$3A_2$,3m,$4A_{\bar{3}}$
	432	$3A_4$,$4A_3$,$6A_2$
	$\bar{4}$ 3m	$3A_{\bar{4}}$,$4A_3$,6m
	4/m$\bar{3}$2/m	$3A_4$,$4A_{\bar{3}}$,$6A_2$,9m

(Klein and Hurlbut. 1977)

Table 2

TRICLINIC-ceramic models				
Crystal class	Symmetry	Model #	Name	# of models
$\bar{1}$	$\bar{1}$	218		13

Table 3

MONOCLINIC-ceramic models				
Crystal class	Symmetry	Model #	Name	# of models
2/m	$\bar{1}$,1A_2,1m	191	gypsum crystal	17
		192	gypsum crystal	10
		193	gypsum crystal	14

Table 4

ORTHORHOMBIC-ceramic models				
Crystal class	Symmetry	Model #	Name	# of models
2/m2/m2/m	$\bar{1}$,3A_2,3m	147	rhombic dipyramid	13
		148	rhombic prism-pinacoid	12
mm2	2m,1A_2	161		14

Table 5

TETRAGONAL-ceramic models				
Crystal class	Symmetry	Model #	Name	# of models
4/m2/m2/m	$\bar{1}$,1A_4,4A_2,5m	45	tetragonal dipyramid	17
		47	tetragonal dipyramid	14
		52		14
		54		9
		55		14
		58		19
$\bar{4}$ 2m	1$A_{\bar{4}}$,2A_2,2m	79	tetragonal scalenohedron	15

Table 6

HEXAGONAL-ceramic models				
Crystal class	Symmetry	Model #	Name	# of models
6/m2/m2/m	$\bar{1}$,1A_6,6A_2,7m	83	hexagonal dipyramid	13
		86	hexagonal prism	16
		97		20
		111	hexagonal prism	16
		137		17
		138	hexagonal dipyramid	22
$\bar{3}$2/m	1$A_{\bar{3}}$,3A_2,3m	102	rhombohedron	16
		103	rhombohedron	10
		104	scalenohedron	11
		105	rhombohedron	11
		106	rhombohedron	17
		107		17
		108	calcite crystal	10
		109	calcite crystal	16
		110		14
		112	calcite crystal	21
		114		18
		116	calcite crystal	11
		117	calcite crystal	4
		118	calcite crystal	18
32	1A_3,3A_2	140	quartz crystal	16
		141	quartz crystal	11

Table 7

ISOMETRIC-ceramic models				
Crystal class	Symmetry	Model #	Name	# of models
4/m$\bar{3}$2/m	3A_4,4$A_{\bar{3}}$,6A_2,9m	1	cube	6
		2	octahedron	18
		14	trapezohedron	11
		17	cube-octahedron-hexoctahedron	6
		22	hexoctahedron	7
		46	octahedron	16
$\bar{4}$3m	3$A_{\bar{4}}$,4A_3,6m	39	tetrahedron-combination of + and -	9
		77	tetrahedron	7

Table 8

TWINS-ceramic models		
Crystal form	Model #	# of models
gypsum	194	14
staurolite {031}	176	19
staurolite {231}	177	17
calcite	115	19
rutile	113	19

Table 9

TRICLINIC-wooden models			
Crystal class	Symmetry	Model #	Name
$\bar{1}$	$\bar{1}$	218	
		0	chalcanthite

Table 10

MONOCLINIC-wooden models			
Crystal class	Symmetry	Model #	Name
2/m	$\bar{1}$,1A_2,1m	195	orthoclase crystal
		197	orthoclase crystal
		201	clinopyroxine crystal
		0	
2	1A_2	0	
		0	

Table 11

ORTHORHOMBIC-wooden models			
Crystal class	Symmetry	Model #	Name
2/m2/m2/m	$\bar{1}$,3A_2,3m	158	
		173	
		0	
		0	
mm2	2m,1A_2	91	
		171	

Table 12

TETRAGONAL-wooden models			
Crystal class	Symmetry	Model #	Name
4/m2/m2/m	$\bar{1}$,1A$_4$,4A$_2$,5m	47	tetragonal dipyramid
		49	tetragonal dipyramid
		57	zircon crystal
		68	zircon crystal
		70	apophyllite crystal
		0	apophyllite crystal
		0	zircon crystal
		0	
$\bar{4}$2m	1A$_4$,2A$_2$,2m	0	tetragonal scalenohedron
		82	tetragonal disphenoid-tetragonal scalenohedron

Table 13

HEXAGONAL-wooden models			
Crystal class	Symmetry	Model #	Name
6/m2/m2/m	$\bar{1}$,1A$_6$,6A$_2$,7m	84	hexagonal dipyramid
		86	hexagonal prism
$\bar{6}$m2	1A$_3$,3A$_2$,3m	0	ditrigonal dipyramid
		0	ditrigonal prism
		102	rhombohedron
$\bar{3}$2/m	1A$_3$,3A$_2$,3m	106	rhombohedron
		109	calcite crystal
		114	
		116	calcite crystal
		121	
		124	
		139	
		144	
		0	
		0	

Table 14

ISOMETRIC-wooden models			
Crystal class	Symmetry	Model #	Name
$4/m\bar{3}2/m$	$3A_4,4A_{\bar{3}},6A_2,9m$	3	
		6	dodecahedron
		7	cube
		10	cube-octahedron-dodecahedron
		10	octahedron
		14	trapezohedron
		15	dodecahedron-trapezohedron
		16	cube-octahedron-hexoctahedron
		17	cube-octahedron-hexoctahedron
		18	trapezohedron
		20	tetrahedron
		21	
		22	hexoctahedron
		0	cube
		0	octahedron-dodecahedron
$\bar{4}3m$	$3A_{\bar{4}},4A_3,6m$	34	tetrahedron
		35	cube-tetrahedron
		36	tristehedron
		37	deltoid dodecahedron
		39	tetrahedron-combination of + and -
		40	cube-tetrahedron
		41	tetrahedron
		41	cube-tetrahedron
		42	tetrahedron-dodecahedron
		43	dodecahedron-cube-tetrahedron
$2/m\bar{3}$	$3A_2,3m,4A_{\bar{3}}$	25	pyritohedron
		26	diploid
		27	cube-pyritohedron
		28	octahedron-pyritohedron
		31	pyritohedron-cube-octahedron
		0	

Table 15

TWINS-wooden models		
Crystal form	Model #	# of models
gypsum	194	1
staurolite {031}	0	1
staurolite {231}	0	3
	0	1
	0	1
rutile	84	2
orthoclase	0	2
aragonite	0	1
pyrite penetration twin	33	1
tetrahedron penetration twin	0	1
octahedron	0	1

Bibliography

Klein, C. and C.S. Hurlbut, Jr., 1977, *Manual of Mineralogy*. 20th edition. John Wiley & Sons, New York, 596pp.